

Multiperiod Asset Pricing

Lorenzo Naranjo

March 2026

Introduction

The [consumption-based asset pricing](#) notebook derived the stochastic discount factor in a two-date economy and showed that the price of any payoff x satisfies $p = E(mx)$, where $m = \beta u'(c_1)/u'(c_0)$. That framework handles one-period claims but leaves open the question of how to price assets whose cash flows arrive over multiple future periods — equities that pay dividends for many years, bonds that mature at a fixed horizon, or options with deferred payoffs.

This notebook extends the framework to multiple periods. The key step is to combine the one-period Euler equation with the law of iterated expectations. The argument is iterative: the price today equals the expected discounted price next period, next period's price equals the expected discounted price the period after, and so on. Each step introduces one additional one-period SDF, and the resulting compounded discount factor $m_{t,t+j}$ — a product of j consecutive one-period SDFs — prices any payoff received at date $t + j$.

Under CRRA utility, the compounded SDF takes the form $\beta^j (c_{t+j}/c_t)^{-\gamma}$, driven entirely by cumulative consumption growth between the pricing date and the payment date. This telescoping is a direct consequence of the power form of marginal utility and is the central result of the notebook. All subsequent pricing formulas for bonds and equities with multiple cash flows follow from it. The [intertemporal portfolio choice](#) notebook extends the framework to allow for time-varying investment opportunities, and [recursive preferences](#) replaces time-separable utility with Epstein-Zin preferences.

Time-Separable Utility

Consider a stream of consumption $\{c_{t+j}\}_{j=0}^{\infty}$. We extend the two-date utility function to infinitely many periods using time-separable expected utility:

$$V_t = E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}). \quad (1)$$

Writing out the first term and re-indexing the remaining sum:

$$\begin{aligned} V_t &= u(c_t) + \beta E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+1+j}) \\ &= u(c_t) + \beta E_t E_{t+1} \sum_{j=0}^{\infty} \beta^j u(c_{t+1+j}) \\ &= u(c_t) + \beta E_t V_{t+1}, \end{aligned} \quad (2)$$

where the second equality uses the law of iterated expectations. Equation (2) is the Bellman equation for (1): it compresses the infinite-horizon problem into a sequence of identical one-period decisions.

The same perturbation argument used in [consumption-based asset pricing](#) applies at every date t . A marginal deviation from an optimal plan — purchasing ξ additional units of any traded asset and reversing the position one period later — must have zero first-order utility effect at the optimum. This yields the one-period Euler equation at every date t :

$$E_t[m_{t,t+1} R_{t+1}] = 1, \quad m_{t,t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}, \quad (3)$$

where R_{t+1} is the gross return on any traded asset between t and $t + 1$.

The Chain of Stochastic Discount Factors

Consider an asset that pays a single dividend D_{t+j} at date $t + j$ and nothing before then. At date $t + j - 1$ the asset has one period remaining, so its price equals

$$p_{t+j-1} = E_{t+j-1}[m_{t+j-1,t+j} D_{t+j}].$$

At date $t + j - 2$ the asset pays nothing and delivers price p_{t+j-1} at the next date. Applying the one-period pricing equation:

$$p_{t+j-2} = E_{t+j-2}[m_{t+j-2,t+j-1} p_{t+j-1}] = E_{t+j-2}[m_{t+j-2,t+j-1} E_{t+j-1}[m_{t+j-1,t+j} D_{t+j}]] = E_{t+j-2}[m_{t+j-2,t+j} D_{t+j}]$$

where the last equality uses the law of iterated expectations. Iterating this argument back to date t gives

$$p_t = E_t[m_{t,t+1} m_{t+1,t+2} \cdots m_{t+j-1,t+j} D_{t+j}].$$

This motivates defining the compounded SDF from t to $t + j$ as the product of j consecutive one-period discount factors:

$$m_{t,t+j} = \prod_{i=1}^j m_{t+i-1,t+i} = \prod_{i=1}^j \beta \frac{u'(c_{t+i})}{u'(c_{t+i-1})}. \quad (4)$$

The ratios of marginal utilities telescope:

$$m_{t,t+j} = \beta^j \cdot \frac{u'(c_{t+1})}{u'(c_t)} \cdot \frac{u'(c_{t+2})}{u'(c_{t+1})} \cdots \frac{u'(c_{t+j})}{u'(c_{t+j-1})} = \beta^j \frac{u'(c_{t+j})}{u'(c_t)}. \quad (5)$$

For an asset with a stream of dividends $\{D_{t+j}\}_{j=1}^{\infty}$, adding up the present value of each cash flow gives the general multiperiod pricing formula.

Property 1 (Multiperiod Pricing Formula). *Under time-separable utility, the price of any asset with dividend stream $\{D_{t+j}\}_{j=1}^{\infty}$ is*

$$p_t = E_t \sum_{j=1}^{\infty} m_{t,t+j} D_{t+j}, \quad m_{t,t+j} = \beta^j \frac{u'(c_{t+j})}{u'(c_t)}. \quad (6)$$

Each dividend is discounted by the compounded SDF for its horizon. No single discount rate is assumed to apply uniformly across maturities; the SDF adjusts both the time value of money and the risk compensation horizon by horizon.

CRRA Utility

Under power utility $u(c) = c^{1-\gamma}/(1-\gamma)$, marginal utility is $u'(c) = c^{-\gamma}$. Substituting into (5):

$$m_{t,t+j} = \beta^j \frac{c_{t+j}^{-\gamma}}{c_t^{-\gamma}} = \beta^j \left(\frac{c_{t+j}}{c_t} \right)^{-\gamma}. \quad (7)$$

The compounded SDF depends on just two quantities: the consumption ratio between the payment date and the pricing date, and the number of periods elapsed. States in which consumption is low at $t+j$ relative to t receive large values of $m_{t,t+j}$, making assets that pay off in sustained downturns especially valuable. Note also that (7) follows directly from the telescoping in (5) without any additional assumptions; the power form of marginal utility causes intermediate consumption levels to cancel exactly.

Pricing Applications

Real Discount Bonds

A real discount bond maturing at horizon T pays one unit of consumption at date $t+T$. Its price is

$$B_t^T = E_t[m_{t,t+T}] = E_t \left[\beta^T \left(\frac{c_{t+T}}{c_t} \right)^{-\gamma} \right]. \quad (8)$$

The bond price equals the expected compounded SDF at horizon T . Since the SDF is large when consumption is low, bonds are valuable insurance against sustained consumption declines, and their prices embed the market's assessment of downside risk over the full maturity. The gross yield over T periods is $1/B_t^T$, and the annualized yield is $(1/B_t^T)^{1/T} - 1$.

Assets with Multiple Cash Flows

An asset paying dividends $\{D_{t+j}\}_{j=1}^T$ is priced by (6) as

$$p_t = \sum_{j=1}^T E_t \left[\beta^j \left(\frac{c_{t+j}}{c_t} \right)^{-\gamma} D_{t+j} \right]. \quad (9)$$

Each term can be decomposed using $E_t(m_{t,t+j} D_{t+j}) = B_t^j E_t(D_{t+j}) + \text{Cov}_t(m_{t,t+j}, D_{t+j})$, separating the time value of money from the covariance risk adjustment. Dividends that co-move positively with consumption growth receive a downward price adjustment because the SDF is low precisely when those dividends are high.

Example 1. Consider an economy with current consumption $c_0 = 1.00$, risk aversion $\gamma = 2$, and discount factor $\beta = 0.95$. Three scenarios — Boom, Normal, and Recession — have probabilities, consumption at dates 1 and 2, and dividends at both dates given by

$$q = \begin{bmatrix} 0.25 \\ 0.50 \\ 0.25 \end{bmatrix}, \quad c_1 = \begin{bmatrix} 1.08 \\ 1.02 \\ 0.94 \end{bmatrix}, \quad c_2 = \begin{bmatrix} 1.16 \\ 1.04 \\ 0.88 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1.20 \\ 0.90 \\ 0.60 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1.35 \\ 0.95 \\ 0.50 \end{bmatrix}.$$

The one-period SDFs $m_{0,1} = 0.95 c_1^{-2}$ and the two-period SDFs $m_{0,2} = 0.95^2 c_2^{-2} = 0.9025 c_2^{-2}$ are

$$m_{0,1} = \begin{bmatrix} 0.814 \\ 0.913 \\ 1.075 \end{bmatrix}, \quad m_{0,2} = \begin{bmatrix} 0.671 \\ 0.834 \\ 1.166 \end{bmatrix}.$$

The one-period and two-period real bond prices are

$$B^1 = \sum_s q(s) m_{0,1}(s) = 0.25(0.814) + 0.50(0.913) + 0.25(1.075) = 0.929,$$

$$B^2 = \sum_s q(s) m_{0,2}(s) = 0.25(0.671) + 0.50(0.834) + 0.25(1.166) = 0.876.$$

The annualized yields are $1/B^1 - 1 = 7.6\%$ and $(1/B^2)^{1/2} - 1 = 6.8\%$.

Using (9), the price of the dividend-paying asset is

$$p = \sum_s q(s) m_{0,1}(s) D_1(s) + \sum_s q(s) m_{0,2}(s) D_2(s).$$

The first sum is $0.25(0.814 \times 1.20) + 0.50(0.913 \times 0.90) + 0.25(1.075 \times 0.60) = 0.817$, and the second is $0.25(0.671 \times 1.35) + 0.50(0.834 \times 0.95) + 0.25(1.166 \times 0.50) = 0.769$. Therefore $p = 0.817 + 0.769 = 1.586$.

For comparison, discounting expected dividends at the risk-free bond prices gives $E(D_1)B^1 + E(D_2)B^2 = 0.90 \times 0.929 + 0.9375 \times 0.876 = 0.836 + 0.821 = 1.657$. The risky asset trades below this value because its dividends co-move positively with consumption: they are highest in the Boom when m is low and lowest in the Recession when m is high. $\square$

Nominal Assets and Derivatives

The same compounded SDF extends to nominal payoffs by adjusting for inflation. Let Π_t denote the price level; then $m_{t,t+j}^* = m_{t,t+j} \Pi_t / \Pi_{t+j}$ is the nominal compounded SDF. The price of a nominal discount bond maturing at T with face value \$1 is

$$B_t^{*T} = E_t \left[m_{t,t+T} \frac{\Pi_t}{\Pi_{t+T}} \right],$$

and the price of a call option with strike K and maturity T written on a nominal asset S_{t+T} is

$$c_t = E_t [m_{t,t+T}^* (S_{t+T} - K)^+].$$

The consumption-based pricing kernel therefore extends from one-period claims to bonds, equities with multiple cash flows, and derivatives across any horizon without any change in the underlying economics.

References