Epstein-Zin Preferences

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Introduction

The consumption and portfolio choice notebook solved the two-date savings problem under CRRA utility. That framework is analytically convenient and delivers a clean consumption-based SDF, but it imposes a strong restriction: the coefficient of relative risk aversion and the elasticity of intertemporal substitution are tied together by a single parameter. Economically, this means the investor's willingness to bear risk and willingness to shift consumption across dates cannot be chosen independently.

Epstein-Zin preferences relax exactly that restriction. They keep the recursive structure needed for asset pricing while separating risk aversion, denoted by γ, from the elasticity of intertemporal substitution, denoted by ψ. This matters in applications because portfolio choice is primarily about attitudes toward risk, whereas the consumption-savings decision is primarily about intertemporal substitution. The CRRA model forces these two margins to move together; Epstein-Zin does not.

This notebook introduces Epstein-Zin in the simplest environment where that distinction can be seen clearly: a one-period model with two dates. The goal is not yet to study long-run risk or state-dependent opportunity sets. Instead, the point is to show that the two-date problem remains tractable, that optimal current consumption is still a constant fraction of wealth by homotheticity, and that the separation between γ and ψ appears immediately in the consumption and portfolio decisions.

Preferences

Intertemporal Aggregation Without Risk

Before introducing uncertainty, it is useful to isolate the intertemporal part of the problem. As in the Fisher model, the agent chooses how to trade off consumption today against consumption tomorrow. A convenient way to represent that tradeoff is with a CES aggregator:

$$U(c_0, c_1) = \left[(1-\beta)c_0^{\rho} + \beta c_1^{\rho}\right]^{1/\rho}.$$

Here $(1-\beta)$ and β are weights on the two dates, so they reflect time preference rather than probabilities over states.

To see how ρ is related to the elasticity of intertemporal substitution, first compute the partial derivatives of the CES aggregator. Differentiating $U = [(1-\beta)c_0^{\rho} + \beta c_1^{\rho}]^{1/\rho}$ gives

$$\frac{\partial U}{\partial c_0} = (1-\beta)\, c_0^{\rho-1} U^{1-\rho}, \qquad \frac{\partial U}{\partial c_1} = \beta\, c_1^{\rho-1} U^{1-\rho},$$

so the factor $U^{1-\rho}$ cancels in the ratio. At an optimum the marginal rate of substitution equals the gross interest rate R:

$$\frac{\partial U/\partial c_1}{\partial U/\partial c_0} = \frac{\beta}{1-\beta}\left(\frac{c_1}{c_0}\right)^{\rho-1} = \frac{1}{R}.$$

Taking logs of both sides and differentiating with respect to $\ln R$ gives

$$(\rho - 1)\,\frac{d\ln(c_1/c_0)}{d\ln R} = -1,$$

so

$$\frac{d\ln(c_1/c_0)}{d\ln R} = \frac{1}{1-\rho}.$$

This derivative is the elasticity of intertemporal substitution: it measures the percentage change in the consumption ratio c_1/c_0 for a one-percent increase in the interest rate. When R rises, future consumption becomes cheaper in terms of current consumption forgone, and the agent shifts spending toward the future. A high ψ means the agent is very responsive to this price change; a low ψ means the agent prefers to keep the consumption profile flat regardless of the return. Setting this equal to ψ gives

$$\psi = \frac{1}{1-\rho}.$$

Solving for ρ gives

$$\rho = 1 - \frac{1}{\psi}.$$

This is why the CES aggregator is written with the exponent $1 - 1/\psi$:

$$U(c_0, c_1) = \left[(1-\beta)c_0^{1-1/\psi} + \beta c_1^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}.$$

The elasticity of intertemporal substitution measures how willing the agent is to shift consumption across dates when the intertemporal rate of return changes. If tomorrow becomes relatively more attractive, a high-ψ agent is more willing to give up some current consumption in exchange for more future consumption. A low-ψ agent is less willing to do so.

Adding Risk

Now reintroduce uncertainty. Let W_0 denote initial wealth and let c_0 be current consumption. The agent invests the remaining wealth $W_0 - c_0$ in a portfolio with gross return

$$R^w = \boldsymbol{\alpha}'(\mathbf{R} - R^f \boldsymbol{\iota}) + R^f,$$

so that next-period wealth and consumption are

$$W_1 = R^w(W_0 - c_0), \qquad c_1 = W_1.$$

Epstein-Zin keeps the same CES intertemporal aggregator, but before comparing tomorrow with today it first replaces risky future consumption by a certainty equivalent:

$$\mathrm{CE}_1 = \left(\mathrm{E}\left(c_1^{1-\gamma}\right)\right)^{\frac{1}{1-\gamma}}.$$

This is the sure level of consumption that yields the same expected utility as the lottery c_1

under CRRA utility with risk aversion γ. To see this, note that a CRRA agent with coefficient γ is indifferent between CE_1 for certain and the lottery c_1 whenever $u(\mathrm{CE}_1) = \mathrm{E}[u(c_1)]$, i.e.,

$$\frac{\mathrm{CE}_1^{1-\gamma}}{1-\gamma} = \mathrm{E}\left[\frac{c_1^{1-\gamma}}{1-\gamma}\right],$$

which gives exactly the expression above. The parameter γ therefore controls how much the agent discounts risky future consumption relative to its expected value: higher γ means greater aversion to dispersion in c_1, so CE_1 falls further below $\mathrm{E}(c_1)$.

The agent then aggregates current consumption c_0 and the certainty equivalent CE_1 exactly as in the deterministic CES problem:

$$V(W_0) = \left[(1-\beta)c_0^{1-1/\psi} + \beta \mathrm{CE}_1^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}.$$

Thus risk is evaluated inside the future date using γ, while the tradeoff between today and the future certainty equivalent is governed by ψ. Substituting the expression for CE_1 into this equation, we obtain the standard two-date Epstein-Zin aggregator:

$$V(W_0) = \left[(1-\beta)c_0^{1-1/\psi} + \beta\left(\mathrm{E}\left(c_1^{1-\gamma}\right)\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \tag{1}$$

Note that when $\psi = 1/\gamma$, the same parameter again controls both risk aversion and intertemporal substitution, and Epstein-Zin collapses to standard CRRA utility.

Solving the Two-Date Problem

Homotheticity and the Consumption Rule

The Epstein-Zin aggregator in (1) is homogeneous of degree one in consumption. If wealth is scaled by a constant factor, both c_0 and c_1 scale by the same factor, so the optimal consumption share is independent of wealth. We therefore guess

$$c_0 = kW_0,$$

for some constant $k \in (0,1)$. Then

$$c_1 = (1-k)W_0 R^w.$$

Substituting into the value function gives

$$\begin{aligned} V(W_0) &= \left[(1-\beta)(kW_0)^{1-1/\psi} + \beta\left(\mathsf{E}\left(((1-k)W_0R^w)^{1-\gamma}\right)\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} \\ &= W_0\left[(1-\beta)k^{1-1/\psi} + \beta(1-k)^{1-1/\psi}\left(\mathsf{E}\left((R^w)^{1-\gamma}\right)\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}. \end{aligned}$$

Define the certainty-equivalent portfolio return

$$\mathcal{R}(\boldsymbol{\alpha}) = \left(\mathsf{E}\left((R^w)^{1-\gamma}\right)\right)^{\frac{1}{1-\gamma}}.$$

Then the value function becomes

$$V(W_0) = W_0\left[(1-\beta)k^{1-1/\psi} + \beta(1-k)^{1-1/\psi}\mathcal{R}(\boldsymbol{\alpha})^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}. \tag{2}$$

The scalar W_0 factors out completely, confirming that k is indeed constant.

Optimal Consumption

For a fixed portfolio $\boldsymbol{\alpha}$, maximizing $V(W_0)$ is equivalent to maximizing the expression inside the brackets, because the outer power in (2) is monotone:

$$(1-\beta)k^{1-1/\psi} + \beta(1-k)^{1-1/\psi}\mathcal{R}^{1-1/\psi},$$

where $\mathcal{R} = \mathcal{R}(\boldsymbol{\alpha})$ is treated as given.

Differentiating with respect to k gives

$$(1-\beta)\left(1-\frac{1}{\psi}\right)k^{-1/\psi} = \beta\left(1-\frac{1}{\psi}\right)(1-k)^{-1/\psi}\mathcal{R}^{1-1/\psi}.$$

After canceling the common factor $1 - 1/\psi$, we obtain

$$(1-\beta)k^{-1/\psi} = \beta(1-k)^{-1/\psi}\mathcal{R}^{1-1/\psi}.$$

Rearranging,

$$\left(\frac{1-k}{k}\right)^{1/\psi} = \frac{\beta}{1-\beta}\mathcal{R}^{1-1/\psi},$$

so

$$\frac{1-k}{k} = \left(\frac{\beta}{1-\beta}\right)^{\psi}\mathcal{R}^{\psi-1}.$$

Therefore the optimal consumption share is

$$k = \frac{1}{1+\left(\frac{\beta}{1-\beta}\right)^{\psi}\mathcal{R}^{\psi-1}}. \tag{3}$$

Thus optimal current consumption is

$$c_0 = \frac{W_0}{1+\left(\frac{\beta}{1-\beta}\right)^{\psi}\mathcal{R}^{\psi-1}},$$

and next-period consumption is

$$c_1 = \frac{\left(\frac{\beta}{1-\beta}\right)^{\psi}\mathcal{R}^{\psi-1}}{1+\left(\frac{\beta}{1-\beta}\right)^{\psi}\mathcal{R}^{\psi-1}}W_0R^w.$$

As in the CRRA case, consumption growth is proportional to the portfolio return:

$$\frac{c_1}{c_0} = \frac{1-k}{k}R^w. \tag{4}$$

This proportionality is a standard implication of homothetic recursive utility in the two-date problem. The homothetic structure goes back to Epstein and Zin (1989) and Weil (1990), while the broader separation between the consumption-wealth ratio and portfolio choice under Epstein-Zin preferences is central to the dynamic portfolio-choice analysis of Campbell and Viceira (1999).

Portfolio Choice

The separation between consumption-savings and portfolio choice is especially transparent in the two-date problem. Using (2), observe that for a fixed consumption share k, the only term that depends on the portfolio choice $\boldsymbol{\alpha}$ is $\mathcal{R}(\boldsymbol{\alpha})$. All other terms are constants from the perspective of the portfolio problem. Therefore the optimal portfolio solves

$$\max_{\boldsymbol{\alpha}} \mathcal{R}(\boldsymbol{\alpha}) = \max_{\boldsymbol{\alpha}} \left(\mathsf{E}\left((R^w)^{1-\gamma} \right) \right)^{\frac{1}{1-\gamma}} . \tag{5}$$

Equation (5) therefore shows that the investor chooses the portfolio that delivers the highest certainty-equivalent return. Thus, in this two-date environment, the risky portfolio depends on risk aversion γ but not on the EIS ψ. The EIS affects only the consumption share k.

Pricing Implications

Stochastic Discount Factor

The Epstein-Zin stochastic discount factor takes the form

$$m = \beta^{\theta} \left(\frac{c_1}{c_0} \right)^{-\theta/\psi} (R^w)^{\theta-1}, \qquad \theta = \frac{1-\gamma}{1-1/\psi}.$$

Using (4),

$$m = \beta^{\theta} \left(\frac{1-k}{k} R^w \right)^{-\theta/\psi} (R^w)^{\theta-1} = \beta^{\theta} \left(\frac{1-k}{k} \right)^{-\theta/\psi} (R^w)^{\theta-1-\theta/\psi}.$$

Since

$$\theta - 1 - \frac{\theta}{\psi} = -\gamma,$$

the SDF simplifies to

$$m = \beta^{\theta} \left(\frac{1-k}{k} \right)^{-\theta/\psi} (R^w)^{-\gamma}. \tag{6}$$

The key point is that, in this two-date setting, the state dependence of the SDF is still just a power of the portfolio return, $(R^w)^{-\gamma}$, exactly as in the CRRA case. The difference is that the constant in front depends on both γ and ψ through the optimal savings rate k.

Relation to CRRA Utility

When $\psi = 1/\gamma$, we have $\theta = 1$ and Epstein-Zin preferences reduce to CRRA utility. In that case,

$$m = \beta \left(\frac{c_1}{c_0} \right)^{-\gamma},$$

and (6) becomes

$$m = \beta \left(\frac{1-k}{k} \right)^{-\gamma} (R^w)^{-\gamma},$$

which matches the result derived in the consumption and portfolio choice notebook.

The new feature of Epstein-Zin is not that the two-date problem loses tractability. Rather, it is that the consumption share depends on ψ while the risky portfolio depends on γ. This is the simplest setting in which the separation of intertemporal substitution from risk aversion becomes visible.

References

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