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# Options on Futures

## Definitions

The underlying asset is a futures contract, usually on a commodity, a precious metal, a currency, an interest rate or a bond. These options usually expire on or a few days before the earliest delivery date of the underlying futures contract. If a call futures option is exercised, the buyer gets a long position in the futures plus a cash amount equal to the excess of the futures price at the time of the most recent settlement over the strike price. If a put futures option is exercised, the buyer acquires a short position in the futures plus a cash amount equal to the excess of the strike price over the futures price at the time of the most recent settlement.

**Example 1.** A September call option contract on copper futures has a strike of 425 cents per pound. A CME contract is written on 25,000 pounds of copper. The call is exercised when the futures price is 426.95 cents and the most recent settlement is 426.45 cents. The trader receives a long September futures contract on copper and $25,000 \times (4.2645 - 4.2500) = \$362.50$. □

**Example 2.** A September put option contract on soybean futures has a strike price of 1,380 cents per bushel. A CME contract is written over 5,000 bushels. The put is exercised when the futures price is 1372 cents per bushel and the most recent settlement price is 1365 cents per bushel. The trader receives a short September futures contract on soybean and $5,000 \times (13.80 - 13.65) = \$750$ in cash. □

For commodities, the futures contract is what trades: as we saw in The Spot Price, the spot price of a commodity is the price of the nearby futures. Exercising a futures option delivers a futures position, which can be closed or rolled easily, not the physical good. Futures options and futures usually trade side by side on the same exchange, such as the Chicago Mercantile Exchange (CME), so futures options are liquid and may entail lower transaction costs.

## Replicating a Futures Option

As in The Black-Scholes Model, take the point of view of a trading desk whose sales team just sold a European option $H$ with maturity $T$ written on a futures contract that expires at $T_F \geq T$. Let $F_t$ denote the futures price at time $t$, and assume that it follows a GBM with volatility $\sigma$. The traders replicate the option by holding $N_F$ futures contracts and $N_Z$ units of a bond.

The difference with a stock is that entering a futures contract costs nothing. The futures position has zero value at all times, because it is marked to market: over each short interval $\Delta t$, a position of $N_{F,t}$ contracts pays $N_{F,t}\Delta F_t$ into the margin account, which earns the risk-free rate. The value of the replicating portfolio is therefore all in the bond,

$$V_t = N_{Z,t} Z_t,$$

and its change over $\Delta t$ is

$$\Delta V_t = N_{F,t}\Delta F_t + N_{Z,t}\Delta Z_t.$$

As $\Delta t \to 0$, and using $dZ = rZdt$,

$$dV = N_F dF + rVdt. \tag{1}$$

The option is a function $V(F,t)$ of the futures price, and Itô's lemma gives the dynamics of the short position:

$$dV = \left(\frac{1}{2}\sigma^2 F^2 \frac{\partial^2 V}{\partial F^2} + \frac{\partial V}{\partial t}\right)dt + \frac{\partial V}{\partial F}dF. \tag{2}$$

Matching (1) and (2), the hedge works if the desk holds

$$N_F = \frac{\partial V}{\partial F}$$

futures contracts, and the option value satisfies

$$\frac{1}{2}\sigma^2 F^2 \frac{\partial^2 V}{\partial F^2} + \frac{\partial V}{\partial t} - rV = 0. \tag{3}$$

Compared with the PDE for an asset paying a dividend yield $\delta$ in Options on Assets Paying a Dividend Yield, the term $(r-\delta)S\,\partial V/\partial S$ is missing. Equation (3) is that PDE with $\delta = r$: a futures price behaves *as if* it were an asset paying a dividend yield equal to the risk-free rate.

## The Risk-Neutral Process for the Futures Price

The replication argument does not depend on the drift of the futures price, so we can price the option in a risk-neutral economy. A futures position costs nothing to enter and pays $dF$, so in that economy its expected gain must be zero, otherwise risk-neutral investors would take unlimited positions. The futures price is therefore a martingale under the risk-neutral measure:

$$dF = \sigma F dB^*.$$

This is the risk-neutral process of an asset with dividend yield $\delta = r$, whose drift is $r - \delta = 0$. Cox et al. (1981) show that, even with random interest rates, the futures price is the risk-adjusted expected spot price at maturity, and Shreve (2004, Theorem 5.6.5) derives the martingale property from the zero value of a futures position. The price of a call is its expected payoff discounted at the risk-free rate,

$$C = e^{-rT}\mathrm{E}^*\big((F_T - K)\mathbb{1}_{\{F_T > K\}}\big),$$

which is the formula for a call on an asset paying a dividend yield, with $F$ in place of $S$ and $r$ in place of $\delta$. The result is known as Black's model. Black (1976) derived it for a European option on a commodity, hedged with the futures contract that matures on the same date as the option, the case we return to in Black's Model in Practice.

Black's Model for European Futures Options

Consider a futures price $F$ that follows a GBM under the risk-neutral measure:

$$dF = \sigma F dB^*.$$

The prices of European call and put futures options with strike price $K$ and time-to-maturity $T$ are given by:

$$C = e^{-rT}[F\ \Phi\ (d_1) - K\ \Phi\ (d_2)],$$
$$P = e^{-rT}[K\ \Phi\ (-d_2) - F\ \Phi\ (-d_1)],$$

where

$$d_1 = \frac{\ln(F/K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}.$$

Put-call parity reads $C - P = e^{-rT}(F - K)$. The delta of the call is $e^{-rT}\ \Phi(d_1)$ futures contracts, whereas the delta of the put is $-e^{-rT}\ \Phi(-d_1)$.

The volatility $\sigma$ is the volatility of the futures price, which is what the desk trades to hedge.

## Black's Model in Practice

When the option and the futures contract mature on the same date, the futures price at maturity is the price of the underlying, $F_T = S_T$. A European option on the futures is then a European option on the underlying, and Black's model prices it from the futures price.

For financial underlyings the futures price is tied to the cash price by carry, $F = Se^{(r-\delta)T}$, and substituting it into Black's model gives back the formulas of Options on Assets Paying a Dividend Yield. Currency traders often prefer the forward form anyway, with $F = Se^{(r-r_f)T}$, because forward rates are quoted directly in the market as forward points, while the interest rate $r_f$ embedded in them need not be observable on its own, as for non-deliverable forwards.

For commodities there is no such substitution. The futures price is not tied to the cash price of the physical good, so it is the input to the formula, and options on commodities are priced off the futures curve directly.

**Example 3.** Consider a 6-month European call option on gold whose payoff is set by the price of gold in six months, when the 6-month futures contract expires. The 6-month futures price is \$1,806, the 6-month risk-free rate is 1% per year continuously-compounded, the strike price is \$1,820, and the volatility of the futures price is 20% per year. The option is priced using Black's model with $F = 1806$, $K = 1820$, $r = 0.01$, and $\sigma = 0.20$:

$$d_1 = \frac{\ln(1806/1820) + \frac{1}{2}(0.20)^2(0.5)}{0.20\sqrt{0.5}} = 0.0161 \Rightarrow \Phi(d_1) = 0.5064$$

$$d_2 = 0.0161 - 0.20\sqrt{0.5} = -0.1253 \Rightarrow \Phi(d_2) = 0.4501$$

$$C = 1806e^{-0.01(0.5)}(0.5064) - 1820e^{-0.01(0.5)}(0.4501) = 94.88$$

The value of the call is \$94.88. To hedge it, a dealer who sold the call buys $e^{-0.01(0.5)}(0.5064) = 0.5039$ futures contracts per unit of the option. □

## Futures Style Options

A futures-style option is a futures contract on a futures option. One of the advantages of these contracts is that the margining of the instrument is the one of a futures contract, which means that you do not have to pay the premium upfront but instead deposit a margin. Gains and losses are then marked to market daily and credited or debited from the margin account.

If we denote by $\phi_C$ the futures price for a call futures-style option and by $C$ the price of the underlying futures option, we have that:

$$\phi_C = Ce^{rT} = F\,\Phi(d_1) - K\,\Phi(d_2)$$

Similarly, the futures price of a put futures-style option is:

$$\phi_P = Pe^{rT} = K\,\Phi(-d_2) - F\,\Phi(-d_1)$$

## Recent Trends

Futures options now trade almost entirely on screens. Floor trading had been fading for years, and in 2021 CME permanently closed most of its remaining pits, including those for grains, keeping only the Eurodollar options pit (Donley 2021). The interest rate complex itself changed underlying: with the end of LIBOR, CME converted the remaining Eurodollar futures and options open interest into SOFR contracts in April 2023 (Mondo Visione 2023), so options on SOFR futures are now the main listed options on short-term US rates.

The largest change is in maturities. Exchanges have added expirations until options on the major futures contracts expire every business day. Options on E-mini S&P 500 futures gained Tuesday and Thursday expirations in 2022, completing daily expirations (CME Group 2022), and Treasury options did the same in June 2025 (CME Group 2025). Grain and oilseed options added Monday through Thursday weeklies in February 2025 to the existing Friday ones. Short-dated commodity options set their fifth consecutive annual volume record in 2025, at an average of about 79,000 contracts a day, more than four times the 2020 level (CME Group 2026).

The demand comes from hedging specific events: a USDA crop report, a Federal Reserve meeting, a tariff announcement, a supply shock in oil. A weekly option covers the few days around the event without paying for months of time value. Activity follows the news, and in March 2026, as the conflict with Iran threatened shipping through the Strait of Hormuz and pushed oil volatility to its highest level since 2022, weekly WTI options traded more than 70,000 contracts a day, three times their 2025 average (CME Group 2026). This use sits uneasily with Black's model, which assumes a constant volatility, whereas an option that expires right after a scheduled announcement is priced mostly on the size of the jump the market expects on that day.

Contract sizes have also shrunk. Micro contracts now carry their own options, from Micro E-mini equity index futures, one tenth the size of the E-mini, to Micro Bitcoin and Micro Ether futures, written on a tenth of one coin, whose options launched in March 2022 (Zacks Equity Research 2022). Together with daily expirations, they let smaller traders hold positions that would be too large, or too long, with the standard contracts.

Bitcoin shows why options follow futures. The desk in Replicating a Futures Option needs only futures and a bond, so once bitcoin futures trade (see Crypto Futures), a dealer can manufacture an option on bitcoin without ever holding a coin: no wallets or keys to secure, a clearinghouse standing behind every trade, and a short position that is as easy to take as a long one. Listed options standardize what dealers could already build. The replication is only as good as the hedge, though, and bitcoin jumps, which leaves a dealer who sold options exposed between rebalancing dates.

## Practice Problems

**Problem 1.** A client asks a trading desk for a European call on a futures contract, with a strike price of 42 and six months to maturity. The futures trades at 40 with a volatility of 25% per year, and the risk-free rate is 4% per year. What price should the desk quote for the call?

**Problem 2.** A refiner wants a European call on the price of WTI crude oil in three months, when the three-month futures contract expires. That futures trades at $72 per barrel with a volatility of 35% per year, the strike is $75, and the risk-free rate is 4% per year. What is the call worth per barrel?

**Problem 3.** A trading desk sells 100 European puts on CME Micro Bitcoin futures, each written on one futures contract for 0.1 bitcoin. The puts expire in one month, when the futures contract expires, and have a strike price of $100,000 per bitcoin. The futures trades at $110,000 per bitcoin with a volatility of 50% per year, and the risk-free rate is 4% per year. What is each put worth, and how many Micro Bitcoin futures does the desk trade to hedge the position?

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