Problem Set 1

Instructions: This problem set is due on 11/19 at 11:59 pm Central Time and is an individual assignment. All problems must be handwritten. Write full sentences to answer questions that require an explanation. Scan your work and submit a PDF file.

Problem 1. You own 200 shares of a stock that trades for $120. Three-month European options on the stock trade at the following prices: a call with strike $130 costs $4.50, and a put with strike $110 costs $3.20.

a. Suppose you write 200 calls against your shares (a covered call). Compute the profit per share at maturity if the stock price ends at $100, $120, $130, and $150. What are the maximum profit for the position and its break-even stock price?
b. Suppose instead you buy 200 puts to protect your shares (a protective put). Compute the profit per share for the same four stock prices. What are the maximum loss for the position and its break-even stock price?
c. Explain which of the two strategies fits an investor who expects the stock to drift up slowly, and which fits an investor who worries about a sharp fall but does not want to give up a large gain.

Problem 2. A trader wants a position with the following payoff at maturity, where S is the stock price at maturity:

$$\text{Payoff} = \begin{cases} 0 & \text{if } S \leq 90, \\ S - 90 & \text{if } 90 < S \leq 100, \\ 10 & \text{if } 100 < S \leq 110, \\ 120 - S & \text{if } 110 < S \leq 120, \\ 0 & \text{if } S > 120. \end{cases}$$

a. Show how to build the payoff using European calls only. State each strike and whether the call is bought or written.
b. Show how to build the same payoff using European puts only.

c. Six-month calls with strikes \$90, \$100, \$110, and \$120 trade for \$18.40, \$11.20, \$6.10, and \$3.00, respectively. Compute the cost of the position built with calls, its maximum profit and maximum loss, and the range of stock prices at maturity for which the trader makes a profit.

Problem 3. A stock trades for \$80 the day before it reports earnings. One-month European options trade at the following prices.

Strike	Call	Put
\$75		\$2.10
\$80	\$3.60	\$3.40
\$85	\$1.70	

a. Compute the cost and the two break-even stock prices of a long straddle with strike \$80.
b. Compute the cost and the two break-even stock prices of a long strangle that buys the put with strike \$75 and the call with strike \$85.
c. For which stock prices at maturity does the straddle earn a higher profit than the strangle?
d. Both strategies bet that the stock will move after the earnings report. Explain why a trader might pay more for the straddle.

Problem 4. The one-year gross return $R = S_1/S_0$ of a stock fund is lognormal, with $\ln(R) \sim \mathcal{N}(0.06, 0.25^2)$.

a. What is the probability that the fund loses money over the year?
b. Compute the median and the mean of R.
c. What is the probability that R ends above its mean? Explain why the answer is below one half.
d. Find the value q such that $\mathrm{P}(R \leq q) = 0.05$. What dollar loss on a \$1,000,000 investment does this value correspond to?

Problem 5. Stocks A and B both trade for \$50 today and follow geometric Brownian motions with the same expected return $\mu = 15\%$ per year. The volatility of stock A is 40% per year and the volatility of stock B is 60% per year.

a. Compute the expected price of each stock ten years from now.

b. For each stock, compute the probability that the price is above \$50 one year from now and ten years from now.
c. Stock B has the same expected price as stock A at every horizon, yet the probability that it ends above \$50 falls as the horizon grows. Explain why.

Problem 6. A leveraged exchange-traded fund promises twice the return of a stock index over each instant. Ignore fees and the cost of borrowing, so that if the index S follows

$$\frac{\mathrm{d}S}{S} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}B,$$

then the value L of the fund follows

$$\frac{\mathrm{d}L}{L} = 2\mu\,\mathrm{d}t + 2\sigma\,\mathrm{d}B.$$

a. Write $\ln(L_T)$ and $\ln(S_T)$ in terms of B_T, and use them to show that

$$\frac{L_T}{L_0} = \left(\frac{S_T}{S_0}\right)^2 e^{-\sigma^2 T}.$$

b. The index has a volatility of 30% per year and ends the year exactly where it started. What is the return of the fund over the year?
c. Explain in a few sentences why an investor who holds the fund for a long time should not expect to earn twice the return of the index.

Problem 7. A stock trades for \$100 and the risk-free rate is 4% per year with continuous compounding. Three-month European options on the stock trade at the following prices.

Strike	Call	Put
\$95	\$8.20	\$2.40
\$105	\$3.10	\$7.05

A *box spread* combines a bull spread with calls, which buys the call with strike \$95 and writes the call with strike \$105, and a bear spread with puts, which buys the put with strike \$105 and writes the put with strike \$95.

a. Show that the box pays \$10 at maturity whatever the stock price.

b. Compute the cost of the box and the continuously compounded interest rate per year that it earns.
c. Describe the trades that exploit the gap between this rate and the risk-free rate, and compute the profit per box at maturity.
d. Leaving the puts aside, what are the lowest and the highest prices the bull spread with calls can have without allowing an arbitrage?

Problem 8. The EUR/USD exchange rate X, the price of one euro in dollars, follows a geometric Brownian motion,

$$\frac{\mathrm{d}X}{X} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}B.$$

A European investor sees the same market through $Y = 1/X$, the price of one dollar in euros.

a. Use Itô's lemma to show that Y also follows a geometric Brownian motion, with

$$\frac{\mathrm{d}Y}{Y} = (\sigma^2 - \mu)\,\mathrm{d}t - \sigma\,\mathrm{d}B.$$

b. Today $X_0 = 1.16$. Suppose that $\mu = 0$ and $\sigma = 8\%$ per year. Compute the expected values of X and of Y one year from now, and the expected values of $\ln(X)$ and $\ln(Y)$.
c. A US investor expects the dollar price of the euro to stay where it is, while a European investor with the same model expects the euro price of the dollar to rise. Explain how both can be right, and why the expected log exchange rates do not have this problem.

Normal Distribution Table

The table below computes $\Phi(z) = \mathrm{P}(Z \leq z)$ if $Z \sim \mathcal{N}(0,1)$ and $z \geq 0$. The rows denote the first decimal whereas the columns denote the second decimal. If you need $\Phi(-z)$, you can always use $\Phi(-z) = 1 - \Phi(z)$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990