Derivative Securities | Prof. Lorenzo Naranjo
FIN 5241 | Fall 2026

Problem Set 3

Instructions: This problem set is due on 12/7 at 11:59 pm Central Time and is an individual assignment. All problems must be handwritten. Write full sentences to answer questions that require an explanation. Scan your work and submit a PDF file.

Problem 1. A non-dividend-paying stock trades for $100 and has a volatility of 20% per year, and the risk-free rate is 4% per year with continuous compounding. Consider two at-the-money European calls with strike $100, one expiring in one month and the other in one year.

a. For each call, compute the delta, the gamma, the theta per calendar day, and the vega per volatility point.
b. Which of the two calls exposes a desk more to large moves in the stock price, and which exposes it more to changes in implied volatility? Explain using your answers to the previous part.

Problem 2. A desk buys 20,000 of the one-month calls of Problem 1 at an implied volatility of 20% and delta hedges them. Over a short interval Δt, the profit of the hedged position is approximately

$$\frac{1}{2}\Gamma S^2\left[\left(\frac{\Delta S}{S}\right)^2 - \sigma^2\Delta t\right]$$

per option. Take one trading day as $\Delta t = 1/252$.

a. Compute the profit of the hedged position over a day in which the stock moves by 2%, and over a day in which it moves by 0.5%.
b. How large must the daily move be for the desk to break even?
c. If implied volatility tends to be higher than the volatility that stocks later deliver, what should the desk expect to earn on average from this position? Who is on the other side of the trade, and why would they accept it?

Problem 3. After a week of selling options to clients, a desk holds a delta-neutral book of options on the stock of Problem 1 with a gamma of $-3{,}000$ and a vega of $-6{,}000$ per volatility point. The desk can trade the two calls of Problem 1 and the stock.

a. Explain why the desk cannot fix both exposures using only one of the two calls.
b. Find the positions in the one-month and the one-year calls that make the book gamma neutral and vega neutral.
c. How many shares of the stock must the desk then buy or sell to restore delta neutrality?

Problem 4. A delta-neutral portfolio of options on a non-dividend-paying stock is worth \$500,000. The stock trades for \$80 and has a volatility of 30% per year, the risk-free rate is 4% per year, and the theta of the portfolio is $-\$40{,}000$ per year.

a. Use the Black-Scholes equation to compute the gamma of the portfolio.
b. Approximately how much does the portfolio gain if the stock jumps by \$2 in either direction, and how much does it lose over one calendar day if the stock does not move?
c. How large must the stock move be over a day for the portfolio to break even?

Problem 5. A bank sells a one-year note on a stock index for \$1,000. At maturity the note pays \$1,000 if the index is above its current level, and \$900 otherwise. The index has a dividend yield of 1.5% per year and a volatility of 18% per year, and the risk-free rate is 4% per year with continuous compounding.

a. Show that the note is a zero-coupon bond plus a cash-or-nothing call, and state the face value of the bond, the payment of the call, and its strike.
b. Compute the value of the note. How much does the bank earn by selling it for \$1,000?
c. A trader hedges the cash-or-nothing call by trading the index. Explain why the hedge is hard to manage when the option is about to expire and the index is close to the strike.

Problem 6. A non-dividend-paying stock trades for \$100. Every month the stock price can increase by 10% or decrease by 10%, and the risk-free rate is 6% per year with continuous compounding. Use a three-step binomial tree to price the following options, which expire in three months and have a strike of \$100.

a. A European call.
b. An up-and-out call with barrier \$120, which pays $\max(S_T - 100, 0)$ at maturity provided the stock price never reaches \$120 or above before or at maturity.

c. The up-and-in call with the same strike and barrier, using your previous answers.
d. An average price Asian call that pays $\max(\bar{S}_T - 100, 0)$, where $\bar{S}_T$ is the arithmetic average of the stock price at months 0, 1, 2, and 3.
e. Explain why the up-and-out call and the Asian call are both cheaper than the European call.

Problem 7. A bank sells a three-year cliquet on a stock index with a notional of \$1,000,000. At the end of each year $i = 1, 2, 3$ the cliquet pays the notional times the return of the index over that year, if the return is positive:

$$1{,}000{,}000 \times \max\left(\frac{S_i}{S_{i-1}} - 1, 0\right).$$

The index has a dividend yield of 2% per year and a volatility of 20% per year, and the risk-free rate is 4% per year with continuous compounding.

a. Show that at the start of year i the payment due at the end of that year is worth the same whatever the level of the index, and compute this value.
b. Discount the three values to today and price the cliquet. Why is the risk-free rate the right discount rate here, while a forward start option on one unit of the index is discounted at the dividend yield?
c. Compare the price of the cliquet with that of a three-year at-the-money call on $1{,}000{,}000/S_0$ units of the index. Explain why the cliquet costs more.

Problem 8. A desk sells a pension fund 1,000 two-year European put contracts on a stock index with strike 5,400, each on 100 units of the index. The index stands at 6,000, its dividend yield is 1.5% and its volatility 18% per year, and the risk-free rate is 4% per year with continuous compounding.

a. Compute the price of one put, the premium the fund pays, and the delta, the vega per volatility point and the rho per percentage point of the put.
b. The next day interest rates rise by 0.5 percentage points and implied volatility rises by 2 points, while the index does not move. Use vega and rho to estimate the change in the value of the desk's position.
c. Explain the sign of rho using the replicating portfolio of the put. What position in two-year zero-coupon bonds, in dollars, makes the desk's position insensitive to small changes in interest rates?

Normal Distribution Table

The table below computes $\Phi(z) = \mathrm{P}(Z \leq z)$ if $Z \sim \mathcal{N}(0,1)$ and $z \geq 0$. The rows denote the first decimal whereas the columns denote the second decimal. If you need $\Phi(-z)$, you can always use $\Phi(-z) = 1 - \Phi(z)$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990