Investment Theory | Prof. Lorenzo Naranjo
FIN 5320 | Fall 2026

Problem Set 3

Instructions: This problem set is due on Wednesday 9/23 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file.

Problem 1. Suppose that there are many stocks in the security market and that the characteristics of stocks A and B are given as follows:

Stock	Expected Return	Standard Deviation
A	10%	5%
B	15%	10%

The correlation between the stock returns is -1. Suppose that it is possible to invest and borrow at the risk-free rate, r_f. What must be the value of the risk-free rate?

Problem 2. Miriam Okafor holds a fully diversified portfolio worth $3,600,000. When she leaves her employer, her deferred compensation plan is settled with $400,000 of Halvorsen Robotics common stock, which she adds to her portfolio. Her financial adviser provides the following forecasts:

Asset	Expected Return	Standard Deviation
Original Portfolio	7.8%	14%
Halvorsen Robotics	16.0%	28%

The correlation between the returns of Halvorsen Robotics and the returns of the original portfolio is 0.45.

a. Miriam is deciding whether to keep the Halvorsen Robotics shares. Assuming she keeps them, calculate the expected return and standard deviation of her new portfolio.
b. If Miriam sells the shares, she will invest the proceeds in Treasury bills yielding 3.5%. Assuming she does so, calculate the expected return and standard deviation of the resulting portfolio.
c. A colleague suggests instead selling the $400,000 of Halvorsen Robotics and buying $400,000 of Quintara Diagnostics, whose expected return and standard deviation are the same as those of Halvorsen Robotics. The argument is that the two stocks are interchangeable, so it makes no difference which one she holds. State whether the colleague is correct or incorrect, and justify your response briefly.

Problem 3. Consider an economy spanned by many risky assets and a risk-free asset that yields 3%. The tangency portfolio (Q) has an expected return of 18% and a standard deviation of 25%. Furthermore, you have information about the following funds:

Fund	Expected Return	Standard Deviation
A	11%	20%
B	7%	16%

The correlation between A and B is 0.25.

a. In a (σ, μ) diagram, draw the capital allocation line (CAL) of Q, and plot funds A and B. Which portfolios are efficient? Why?
b. Aditya wants to invest in an efficient portfolio (i.e. maximum Sharpe ratio) that offers an expected return of 24%. What should he do? Specify in which assets he should invest, and the standard deviation of such portfolio.
c. Rosalind, on the other hand, for regulatory reasons can only invest in funds A and B (and not the risk-free asset). She is aiming for an expected return of 9%. What would you recommend to her? Please clearly indicate the composition and the standard deviation of such portfolio. Is her portfolio efficient?

Problem 4. Two risky assets A and B have the following characteristics:

- $\mathrm{E}(R_A) = 0.10, \sigma_A = 0.10$
- $\mathrm{E}(R_B) = 0.15, \sigma_B = 0.25$
- $\rho_{AB} = +1$

a. Suppose that you aim for an expected return of 5%, what should be the composition of your portfolio? What is the risk of that portfolio? Explain how this strategy can be possible.
b. Draw the investment opportunity set and clearly identify the minimum variance portfolio. Determine its composition and its characteristics.
c. Suppose that in addition to A and B, there is a risk-free asset offering a return of 4%. Is there an arbitrage opportunity? If so, clearly explain how an investor could arbitrage this opportunity (what he buys and what he sells).

Problem 5. Consider an economy spanned by N risky assets and a risk-free asset. The tangency portfolio (Q) has an expected return of 16% and a standard deviation of 20%. You also know that Dmitri chooses to optimally invest in portfolio (P_1) with an expected return of 13% and a standard deviation of 14%.

a. Compute the risk-free rate r_f of this economy.
b. What is the composition of the portfolio owned by Dmitri?
c. Ingrid wants to invest optimally in a portfolio (P_2) that has a standard deviation of 30%. What would you recommend to her? Please clearly indicate the composition of her portfolio, and the expected return that she will be able to achieve.