Prof. Lorenzo Naranjo

Problem Set 4

Instructions: This problem set is due on Monday 10/12 at 11:59 pm CST and is an individual assignment. All problems must be handwritten. Scan your work and submit a PDF file. Treat it as a mock final exam: work through it in 2 hours, closed book, using only the formula sheet and a calculator.

Problem 1.

a. If you use the CAPM as your asset pricing model, can a stock have a positive alpha but a Sharpe ratio lower than the market? How can this be possible?
b. Why do we claim that a stock with a negative alpha is considered overpriced?
c. Can a risky financial asset have no firm-specific risk? If so, give an example of such an asset.
d. In the group project, you maximized the Sharpe ratio of a portfolio of sector ETFs with and without short sales. Why can the short-sale constrained portfolio never have a higher Sharpe ratio than the unconstrained one in the sample used to estimate the inputs? Why might it nevertheless perform better out of sample?

Problem 2. The risk-free rate is 4%, and the market portfolio has an expected return of 10% with a standard deviation of 20%. Harbor Logistics stock has a return standard deviation of 40% and a correlation of 0.35 with the market. Harbor trades at $100 per share and is expected to pay a constant dividend in perpetuity.

a. Compute the beta of Harbor stock and its expected return according to the CAPM.
b. What fraction of the variance of Harbor's returns is systematic? What is the standard deviation of its firm-specific returns?
c. An analyst forecasts that Harbor will earn 12% over the coming year. Compute Harbor's alpha given this forecast. According to the analyst, is the stock underpriced, overpriced, or fairly priced?

d. Ignore the analyst's forecast and assume that investors discount Harbor's dividends at the CAPM expected return from part a, so that the price satisfies the perpetuity formula

$$P = \frac{D}{\mathrm{E}(r)}.$$

Compute the dividend that justifies the current price of $100. What would be the new price of Harbor stock if its correlation with the market doubled, with all other variables unchanged?

Problem 3. Consider the two (excess return) index-model regression results for stocks A and B:

Stock	Alpha	Beta	Firm-Specific Standard Deviation
A	-0.5%	1.4	10%
B	2.5%	0.6	23%

The risk-free rate over the period was 5%, whereas the market's average return was 12% with a standard deviation of 25%.

a. Compute the expected return of stocks A and B.
b. Compute the variance and standard deviation of stocks A and B.
c. Compute the Sharpe ratio of stocks A and B.
d. Compute the regression R-square of stocks A and B.
e. Compute the covariance of returns between A and B.
f. Suppose you form a portfolio P composed of 25% of A and 75% of B. Compute the expected return, beta, standard deviation and firm-specific standard deviation of portfolio P.

Problem 4. A pension fund manager is considering three mutual funds. The first is a stock fund, the second is a long-term government and corporate bond fund, and the third is a T-bill money market fund. The statistical properties of the funds are as follows.

	Expected Return	Standard Deviation
Stock Fund (S)	15%	25%
Bond Fund (B)	8%	15%

	Expected Return	Standard Deviation
T-bill Fund (F)	5%	0%

Analysts estimate that the maximum Sharpe ratio portfolio (Q) has an expected return of 20% and a standard deviation of 30%.

a. In a μ-σ diagram, draw the capital allocation line (CAL) of Q, and indicate where the mutual funds S, B, and F are located. Which portfolios are efficient or inefficient? Why?
b. Determine the weights of an efficient portfolio that has the same standard deviation as the stock fund (S). What's the expected return of this portfolio?
c. Determine the weights of an efficient portfolio that has the same expected return as the bond fund (B). What's the standard deviation of this portfolio?
d. The pension fund ranks portfolios using the utility function $U = \mu - \frac{1}{2}A\sigma^2$ with $A = 5$. What fraction of the fund should be invested in Q, and what fraction in T-bills? Compute the expected return, standard deviation and certainty equivalent of this optimal portfolio, and compare it with the certainty equivalent of investing everything in the stock fund (S).

Problem 5. Next year the economy can be in one of three states. You forecast the following returns for two stocks, X and Y.

State	Probability	Return on X	Return on Y
Boom	0.3	28%	4%
Normal	0.4	12%	18%
Recession	0.3	-8%	-2%

a. Compute the expected return and standard deviation of each stock.
b. Compute the covariance and the correlation between the returns of X and Y.
c. Compute the weights, expected return and standard deviation of the minimum-variance portfolio of X and Y.
d. Compare the minimum-variance portfolio with stock Y. Would a mean-variance investor who can only invest in X and Y ever put all of her wealth in Y? Explain.

e. The weights you found in part c depend only on the variances and the covariance of the two stocks, not on their expected returns. In the group project, you estimated both the maximum Sharpe ratio and the minimum variance portfolios from five years of monthly returns. Explain why not relying on expected returns can be an advantage of the minimum variance portfolio.

Problem 6. Two risky assets A and B have the following characteristics:

- $\mathrm{E}(r_A) = 0.20, \sigma_A = 0.28$
- $\mathrm{E}(r_B) = 0.30, \sigma_B = 0.48$
- $\rho_{AB} = +1$

a. Compute the expected return of an asset with zero return variance.
b. Suppose that in addition to A and B, there is a risk-free asset offering a return of 5%. Is there an arbitrage opportunity? If so, clearly explain how an investor could arbitrage this opportunity, and explain why this would be an arbitrage.

Problem 7. You have a logarithmic utility function $u(W) = \ln W$ and your current wealth is \$10,000. There is a 10% probability that your wealth decreases to \$1,000, and a 90% probability that your wealth remains at \$10,000. You are considering two insurance policies:

- **Plan A**: Pays \$5,000 if your wealth decreases to \$1,000 and costs \$500 upfront.
- **Plan B**: Pays \$8,000 if your wealth decreases to \$1,000 and costs \$800 upfront.

Evaluate whether you should choose Plan A, Plan B, or opt not to purchase insurance. Provide a clear explanation of your decision.